A Novel Method for Ergodic Sum Rate Analysis of Spatial Modulation Systems with Maximum Likelihood Receiver

(Invited Paper)

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***Abstract*—This paper proposes a novel method for ergodic sum rate analysis of spatial modulation (SM) systems with maximum likelihood receiver. This method is developed based on the M_T–ary symmetric channel, where M_T is the number of transmit antennas. The probability of antenna detection error is approximated by the pair-wise error probability. Then, an approximation to the ergodic sum rate of information transmission via SM with maximum likelihood receiver is computed. It is demonstrated via simulation that the proposed analysis method is able to provide an excellent approximation to the ergodic sum rate of SM.**

***Keywords* – Spatial modulation, M_T–ary symmetric channel, ergodic sum rate bounds.**

I. Introduction

Antenna indices are recently utilized to convey information in new multiple-input multiple-output (MIMO) transmission schemes known as spatial modulation (SM) [1] and space shift keying (SSK) [2], [3]. These schemes activate one antenna only at each transmission instant and are able to provide a number of benefits [1]–[6]. First, inter-channel interference (ICI) can be mitigated as only one antenna is used. Second, inter-antenna synchronization is largely simplified. Third, low cost radio chain systems can be deployed in SM and SSK. Last, receiver complexity can be reduced as compared with the Bell Labs layered space-time (BLAST) scheme. Therefore, SM is regarded as a potential technology for the fifth generation (5G) wireless communication systems [7].

The performance of SM has been extensively discussed in the literature. Bit error rate (BER) performance of SM with perfect channel state information at the receiver (CSIR) was studied in [1], [3], and [8]. In addition to perfect CSIR, the BER performance of SM/SSK in the presence of imperfect CSIR was investigated in [9]–[11]. Although the capacity of a SSK system was investigated in [2], the maximum achievable ergodic sum rates of SM have not been sufficiently investigated in the literature. The authors in [12] discussed the maximum achievable sum rate of SM, provided that continuous Gaussian signals are assumed and a particular channel realization is considered with a single receive antenna and perfect CSIR. Considering a given channel realization, the mutual information between the input and output signal vectors of SM systems was derived and was shown to consist of two main parts. While the first part concerns the information rate due to the radiated (modulated) symbols similar to the case of conventional MIMO systems, the second part involves the additional information conveyed by antenna indices. An extension of [12] from continuous Gaussian source signals to finite alphabet inputs was given in [13]. In this work, the input signals drawn from finite discrete constellations, such as M-ary phase shift keying (M-PSK) modulation, pulse amplitude modulation (PAM) and quadrature amplitude modulation (QAM), are considered. The analysis in this work, however, is still based on a single-antenna receiver and a given channel realization. The maximum achievable sum rate of SM with multiple transmit and receive antennas was later derived in [14]. By appropriately projecting the M_R-dimensional received vector, with M_R being the number of receive antennas, into a 1-dimensional space, a sufficient statistic for detecting the active antenna index was derived and was used for computation of the information rate conveyed by the antenna indices. Lower bounds of the achievable sum rate were also given. Nevertheless, the above analysis was obtained based on a given channel realization. The evaluation of the ergodic achievable sum rate has not been studied in the literature. To fill this research gap, the aim of this paper is to investigate ergodic sum rates and multiplexing gain of SM with perfect CSIR and maximum likelihood detector, with no restrictions on the antenna numbers.

The **contributions** of this paper are listed as below:

1) A novel analysis method is proposed for ergodic sum rate for SM. Especially, the mutual information between the transmitter and receiver via antenna indices is calculated using an M_T-ary symmetric channel. Then, an approximation of ergodic sum rate for SM with maximum likelihood detectors is derived.
2) The probability of antenna detection errors is approximated with pair-wise error probabilities (PEP).

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3) It is demonstrated that the proposed approximation is able to align excellently with simulation results.

The remainder of this paper is organized as follows. Section II gives the general description of the SM system. Section III calculates the ergodic sum rate of SM with the maximum likelihood detector. Results are discussed in Section IV and conclusions are drawn in Section V.

II. System Model

Let us consider a SM system with M_T transmit antennas and M_R receive antennas in an independently and identically distributed (i.i.d) block Rayleigh channel. In this case, $\log_2 M_T$ bits will be carried by antenna indices. Let A be a uniformly distributed random variable on the index set $J = \{1, 2, \cdots, M_T\}$ representing the selected antenna index to transmit SM symbols. In addition, we denote the channel vector between the l-th transmit antenna ($l \in J$) and receive array by $\mathbf{h}_l$. The kth ($k = 1, 2, \cdots, M_R$) entry h_{kl} in $\mathbf{h}_l$ follows the i.i.d unit-variance zero-mean complex Gaussian distribution, i.e., $h_{kl} \sim \mathcal{CN}(0, 1)$. The set L of all channel vectors can be listed as $L = \{\mathbf{h}_1, \mathbf{h}_2, \cdots, \mathbf{h}_{M_T}\}$.

Next, let us define a mapping $g : J \to L$, $g(l) = \mathbf{h}_l, \forall l \in J$, as well as its inverse $g^{-1} : L \to J$, $g^{-1}(\mathbf{h}_l) = l$. Hence, J and L are isomorphic. The mappings g and g^{-1} can be regarded as the encoding and decoding functions for the information conveyed by antenna indices. In addition, the transmitted symbol X is a random variable. The received symbol Y can be obtained as

$$Y = HX + N \tag{1}$$

where N is a random vector whose entries are i.i.d zero-mean complex Gaussian random variables with variance $1/\rho$, ρ is the signal-to-noise ratio (SNR), and H is the encoded antenna index with $H = g(A)$.

Assuming perfect CSIR, the maximum likelihood SM detector in [15] is applied to the detection of antenna index $\hat{H}$ and transmitted symbol $\hat{X}$,

$$\begin{aligned}\left[\hat{H}, \hat{X}\right] &= \underset{H \in L, X}{\arg\max}\, e^{-\frac{\|Y - HX\|_{\mathrm{F}}^2}{2\sigma^2}} \\ &= \underset{H \in L, X}{\arg\min}\, \|Y - HX\|_{\mathrm{F}}^2. \end{aligned} \tag{2}$$

where σ^2 is the noise power and $||\cdot||_{\mathrm{F}}$ is the Frobenius norm.

III. Ergodic Sum Rate Analysis

In this paper, the ergodic sum rate of spatial modulation R_{SM} is defined as the average mutual information between the source and destination. Channel state information is not assumed at the transmitter. Additionally, without loss of generality, bandwidth is assumed to be normalized. From (1), it can be observed that a SM system can be regarded as a noisy multiple access channel.

The ergodic sum rate of spatial modulation R_{SM} can be approximated by

$$\begin{aligned} R_{\mathrm{SM}} &= \mathrm{E}\left[\max I(X, A; Y)\right] \\ &= \mathrm{E}\left[\max I(X; Y|A)\right] + \mathrm{E}\left[\max I(A; Y)\right] \\ &\approx R_1 + R_2 \end{aligned} \tag{3}$$

where

$$R_1 = \mathrm{E}\left[\max I(X; Y|A)\right] \tag{4}$$

$$R_2 = \mathrm{E}\left[\max I(A; Y|X)\right]. \tag{5}$$

Therefore, the ergodic sum rate of SM is approximated as long as R_1 and R_2 are obtained.

A. Calculation of R_1

As J and L are isomorphic, the value of R_1 can be computed via the well-known channel capacity R_{SIMO} of a single-input multiple-output (SIMO) system, when the transmitted symbol X follows the Gaussian distribution, i.e., $X \sim \mathcal{N}(0, 1)$. Then, R_1 can be calculated as

$$\begin{aligned} R_1(\rho) &= \mathrm{E}\left[\max I(X; Y|A)\right] \\ &= \mathrm{E}\left[\max I(X; Y|H)\right] \\ &= \mathrm{E}\left[\frac{1}{M_T}\sum_{l=1}^{M_T} \log_2\left(1 + \rho\|\mathbf{h}_l\|_{\mathrm{F}}^2\right)\right] \\ &= R_{\mathrm{SIMO}}. \end{aligned} \tag{6}$$

Let ξ be the combined power gain defined by $\xi = ||\mathbf{h}_l||_{\mathrm{F}}^2 = \sum_{k=1}^{M_R} |h_{kl}|^2$. Based on the assumption that h_{kl} is an i.i.d unit-variance zero-mean complex Gaussian random variable, ξ obeys the Chi-square distribution with degree $2M_R$. The cumulative distribution function (CDF) $F_{\Xi}(\xi)$ is expressed as

$$F_{\Xi}(\xi) = \frac{1}{(M_R - 1)!}\gamma(M_R, \xi) \tag{7}$$

where $\gamma(\alpha, x) = \int_0^x t^{\alpha-1} e^{-t} dt$ is the incomplete Gamma function [16]. Therefore, R_1 in terms of SNR ρ can be computed as (9) where $\mathrm{Ei}(\cdot)$ is the exponential integral [16] defined by

$$\mathrm{Ei}(-z) = -\int_z^{\infty} \frac{e^{-t}}{t} dt. \tag{8}$$

It can be observed that the ergodic sum rate of SM is larger than or equal to the that of SIMO.

B. Calculation of R_2

The calculation of R_2 is via the mutual information between A and Y conditioned on X. When the transmitted symbol X is given at the receiver, information is solely carried by antenna indices. This part of information can be contaminated by noise. If the decoded value of the antenna index is not equal to the transmitted antenna index, the transmitted information will have errors. The probability of error detection on antenna indices depends on the noise level. Let $p(H \neq \hat{H}|X)$ be

$$
\begin{aligned}
R_1(\rho) &= \int_0^\infty \log_2(1+\rho\xi)dF_\Xi(\xi) = \int_0^\infty \log_2(1+\rho\xi)\frac{1}{(M_R-1)!}\xi^{M_R-1}e^{-\xi}d\xi \\
&= \begin{cases} -\frac{1}{\ln 2}\mathrm{Ei}\left(-\frac{1}{\rho}\right)\exp\left(\frac{1}{\rho}\right) & (M_R=1) \\ \frac{1}{\ln 2}\sum\limits_{m=0}^{M_R-1}\frac{1}{(M_R-1-m)!}\left[\frac{(-1)^{M_R-m}}{\rho^{M_R-1-m}}\mathrm{Ei}\left(-\frac{1}{\rho}\right)\exp\left(\frac{1}{\rho}\right)+\sum\limits_{k=1}^{M_R-1-m}\frac{(k-1)!}{(-\rho)^{M_R-1-m-k}}\right] & (M_R>1), \end{cases}
\end{aligned} \tag{9}
$$

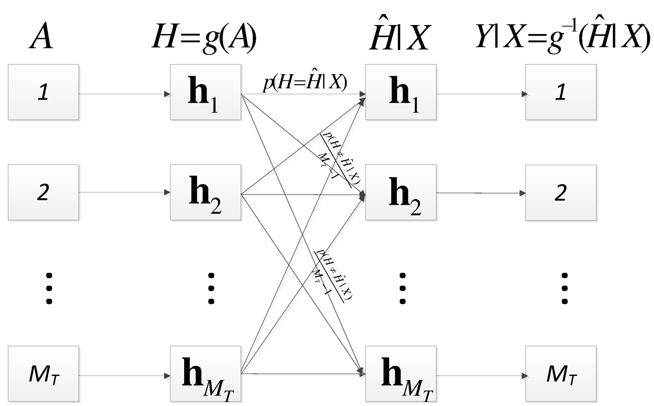

Fig. 1. M_T–ary channel formed by the information transmission via antenna indices.

the probability of antenna detection error given X. Since the channel vectors are i.i.d, we further assume that the probabilities that antenna l is active and antenna q is detected for all $q \neq l$ give X are equal, i.e.,

$$
p(H=\mathbf{h}_l, \hat{H}=\mathbf{h}_q|X) = \frac{p(H\neq\hat{H}|X)}{M_T-1}. \tag{10}
$$

As a result, the information transmission from source A to destination $Y|X$ can be abstracted as an M_T–ary symmetric channel as shown in Fig. 1. First, the source information A carrying $\log_2 M_T$ bits is encoded by g to form H. Second, H is sent through the channel to the destination. Third, given X, the destination attempts recovering $\hat{H}|X$, although part of the information may be corrupted because of noise. Finally, $Y|X$ is decoded via $\hat{H}|X$ by mapping g^{-1}. Also, it should be noted that $Y|X$ and $\hat{H}|X$ are isomorphic.

Channel capacity of an M_T-ary symmetric channel can be found in [17]. Therefore, the ergodic sum rate in an M_T-ary symmetric channel between A and Y given X is calculated as

$$
\begin{aligned}
R_2 = \mathrm{E}\left[\max I(A;Y|X)\right] &= \log_2 M_T - b[p(H\neq\hat{H}|X)] \\
&\quad - p(H\neq\hat{H}|X)\log_2(M_T-1)
\end{aligned} \tag{11}
$$

where $b[\mu] = -\mu\log_2\mu - (1-\mu)\log_2(1-\mu)$ is the binary entropy function.

The calculation of $p(H\neq\hat{H}|X)$ may be difficult. However, the PEP $\mathrm{PEP}(H=\mathbf{h}_l, \hat{H}=\mathbf{h}_q|X=x)$ (transmitted antenna index is l, received antenna index is q, transmitted symbol is x) is easier to obtain [15]

$$
\begin{aligned}
&\mathrm{PEP}(H=\mathbf{h}_l, \hat{H}=\mathbf{h}_q|X=x) \\
&\qquad = \gamma_a^{M_R}\sum_{k=0}^{M_R-1}\binom{M_R-1+k}{k}(1-\gamma_a)^k
\end{aligned} \tag{12}
$$

where $a=\rho x^2$ and $\gamma_a = \frac{1}{2}\left(1-\sqrt{\frac{a}{2+a}}\right)$.

According to the union bound method [18], the probability $p(H\neq\hat{H}|X=x)$ of antenna detection error when $X=x$ should be upper bounded by

$$
p(H\neq\hat{H}|X=x) \leq \sum_{q\neq l}\mathrm{PEP}(H=\mathbf{h}_l, \hat{H}=\mathbf{h}_q|X=x). \tag{13}
$$

However, it was reported in [15] that this upper bound is tight when the SNR is moderate to high ($\rho \geq 10$ dB). Therefore, to simplify the computation, we approximate the probability of antenna detection error given $X=x$ by the sum of PEPs as

$$
\begin{aligned}
&p(H\neq\hat{H}|X=x) \approx \sum_{q\neq l}\mathrm{PEP}(H=\mathbf{h}_l, \hat{H}=\mathbf{h}_q|X=x) \\
&= (M_T-1)\gamma_a^{M_R}\sum_{k=0}^{M_R-1}\binom{M_R-1+k}{k}(1-\gamma_a)^k.
\end{aligned} \tag{14}
$$

Consequently, $p(H\neq\hat{H}|X)$ is approximated by

$$
p(H\neq\hat{H}|X) \approx \int_{-\infty}^{\infty} p(H\neq\hat{H}|X=x)dF_x(x). \tag{15}
$$

With the assumption that the transmitted symbol is Gaussian distributed with zero mean and unit variance, the differentiation of the CDF of X is

$$
dF_X(x) = \frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}dx. \tag{16}
$$

Thus, the integral in (15) can be evaluated via numerical integrations. Finally, R_2 can be computed by combining (11), (14), and (15).

C. High SNR Approximation

In the high SNR regime, the probability of antenna detection error tends to 0, i.e.,

$$
\lim_{\rho\to\infty} P(H\neq\hat{H}|X) \to 0. \tag{17}
$$

Consequently, the high SNR approximation of ergodic sum rate of SM $R_{\mathrm{SM}}^{\mathrm{u}}$ is expressed as [14]

$$\lim_{\rho\to\infty} R_{\mathrm{SM}}^{\mathrm{u}} = \mathrm{E}\left[\log_2(\rho\|\mathbf{h}_l\|_{\mathrm{F}}^2)\right] + \log_2 M_T. \tag{18}$$

Moreover, the multiplexing gain G_{mul} can be defined as the ratio of the sum rate to the sum rate of SISO [19] in the high SNR regime.

$$\begin{aligned} G_{\mathrm{mul}} &= \lim_{\rho\to\infty} \frac{R_{\mathrm{SM}}}{R_{\mathrm{SISO}}} = \lim_{\rho\to\infty} \frac{\int_0^\infty \log_2(1+\rho\xi)dF_\Xi(\xi) + \log_2 M_T}{\int_0^\infty \log_2(1+\rho z)dF_Z(z)} \\ &= \lim_{\rho\to\infty} \frac{\int_0^\infty \log_2(\rho\xi)dF_\Xi(\xi)}{\int_0^\infty \log_2(\rho z)dF_Z(z)} \\ &= \lim_{\rho\to\infty} \frac{\frac{\partial}{\partial\rho}\int_0^\infty \log_2(\rho\xi)dF_\Xi(\xi)}{\frac{\partial}{\partial\rho}\int_0^\infty \log_2(\rho z)dF_Z(z)} \\ &= 1 \end{aligned} \tag{19}$$

where $F_Z(z)$ is the distribution function of exponential distributed random variables. The L' Hospital rule is applied to the above derivations. The multiplexing gain of SM is 1 because SM does not utilize multiple eigen channels. As a result, spatial multiplexing is more appropriate for boosting system ergodic sum rate than SM in the high SNR regime.

D. Ergodic Sum Rate of SSK

By setting $R_1 = 0$, information is only transmitted via antenna indices. In this case, the ergodic sum rate of SM reduces to the ergodic sum rate of SSK. Therefore, the ergodic sum rate R_{SSK} of SSK can be approximated by

$$\begin{aligned} R_{\mathrm{SSK}} \approx R_2 = \log_2 M_T - b[p(H \neq \hat{H}|X)] \\ - p(H \neq \hat{H}|X)\log_2(M_T - 1). \end{aligned} \tag{20}$$

The ergodic sum rate of SSK is upper bounded by $\log_2 M_T$.

IV. Results and Analysis

Fig. 2 shows the probability of antenna detection error in terms of SNR. The approximated probability in (15) is always larger than the simulated probability because of the union bound. It can be observed that the gap between approximation and simulation reduces as SNR increases. The approximation is tight when SNR is larger than 5 dB.

Ergodic sum rates of SM, SIMO, and SISO are compared in Fig. 3. To begin with, both SM and SIMO are able to provide significant improvement in terms of ergodic sum rate than SISO. SM has a higher ergodic sum rate bound than SIMO because of the additional information conveyed by antenna indices. The approximation $\lim_{\rho\to\infty} R_{\mathrm{SM}}^{\mathrm{u}}$ in (18) [14] aligns well with simulation results when SNR is relatively high. However, this approximation fails in the moderate SNR regime. Conversely, it can be observed that the proposed approximation

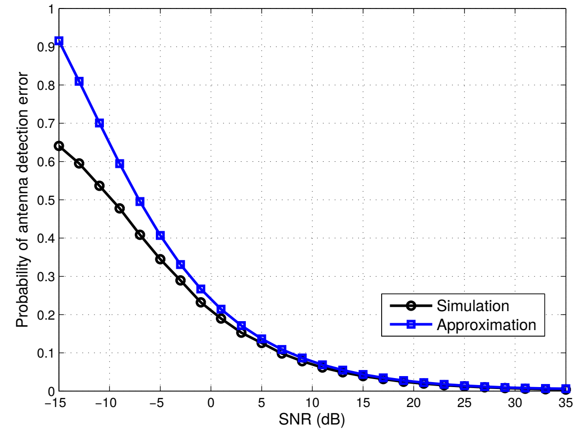

Fig. 2. Probability of antenna detection error ($M_T = 4, M_R = 16$).

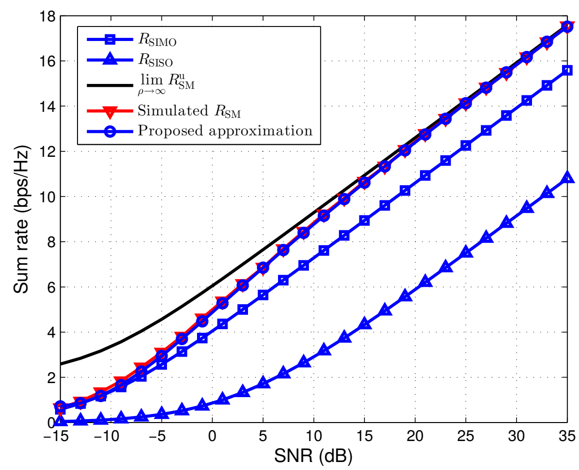

Fig. 3. Comparisons of ergodic sum rate bounds among SM, SIMO, and SISO ($M_T = 4, M_R = 16$).

of ergodic sum rate of SM aligns excellently with simulation results. In the low SNR regime, the proposed approximation of ergodic sum rate of SM approaches SIMO performance. In the high SNR regime, the proposed approximation of ergodic sum rate of SM approaches the approximation $\lim_{\rho\to\infty} R_{\mathrm{SM}}^{\mathrm{u}}$ in [14].

Moreover, ergodic sum rates between SM and SIMO with respect to different receive antenna numbers are shown in Fig. 4. With a fixed number of transmit antennas, the increase of receive antennas results in a boost in ergodic sum rates for both SM and SIMO. However, SM outperforms SIMO by $\log_2 M_T$ bps/Hz for the employment of antenna indices. Additionally, the high SNR approximation $\lim_{\rho\to\infty} R_{\mathrm{SM}}^{\mathrm{u}}$ of SM in (18) [14] will over estimate the ergodic sum rates, while the proposed approximation is tight to simulation results regardless the number of receive antennas.

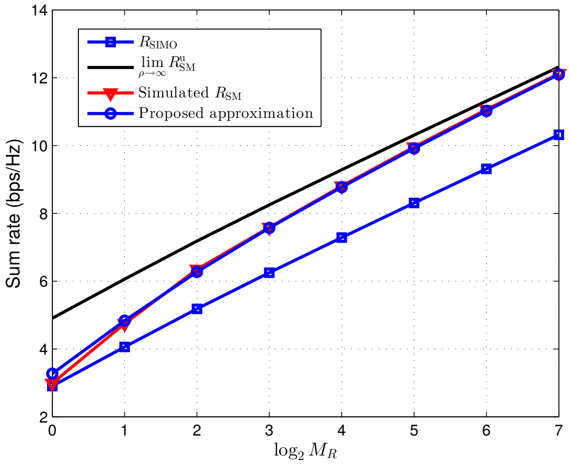

Fig. 4. Comparisons of ergodic sum rate bounds between SM and SIMO with respect to different receive antenna numbers ($M_T = 4, \rho = 10$ dB).

V. Conclusions

This paper has proposed a novel method to analyze ergodic sum rates of SM systems with maximum likelihood receivers. This novel analysis method has divided the ergodic sum rate of a SM system into two parts. The first part is calculated via the ergodic sum rate of SIMO. The second part is calculated via the mutual information in an M_T–ary symmetric channel. The probability of antenna detection error given the transmitted symbol has been approximated by PEPs. It has been demonstrated via simulations that the proposed analysis method is able to provide better approximation than current high SNR approximations in the literature. The proposed approximation aligns excellently with simulation results in both the high and low SNR regimes. In the future work, this analysis method can be applied to calculation ergodic sum rate of SM systems with other detection algorithms such as zero forcing (ZF) algorithms and minimum mean squared error (MMSE) detections. Also, the proposed analysis method can be generalized to spatially correlated channels.

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